Tail Dependence of Factor Models *

Y. Malevergne[1,2] and D. Sornette[1,3]

[1] Laboratoire de Physique de la Matière Condensée CNRS UMR 6622
Université de Nice-Sophia Antipolis, 06108 Nice Cedex 2, France
[2] Institut de Science Financière et d'Assurances - Université Lyon I
43, Bd du 11 Novembre 1918, 69622 Villeurbanne Cedex, France
[3] Institute of Geophysics and Planetary Physics and Department of Earth and Space Science
University of California, Los Angeles, California 90095, USA

email: Yannick.Malevergne@unice.fr and sornette@unice.fr
fax: (33) 4 92 07 67 54

03/02/2002

Abstract

Using the framework of factor models, we establish the general expression of the coefficient of tail dependence between the market and a stock (i.e., the probability that the stock incurs a large loss, assuming that the market has also undergone a large loss) as a function of the parameters of the underlying factor model and of the tail parameters of the distributions of the factor and of the idiosyncratic noise of each stock. Our formula holds for arbitrary marginal distributions and in addition does not require any parameterization of the multivariate distributions of the market and stocks. The determination of the extreme parameter, which is not accessible by a direct statistical inference, is made possible by the measurement of parameters whose estimation involves a significant part of the data with sufficient statistics. Our empirical tests find a good agreement between the calibration of the tail dependence coefficient and the realized large losses over the period from 1962 to 2000. Nevertheless, a bias is detected which suggests the presence of an outlier in the form of the crash of October 1987.

Introduction

The concept of extreme or "tail dependence" probes the reaction of a variable to the realization of another variable when this realization is of extreme amplitude and very low probability. The dependence, and especially the extreme dependence, between two assets or between an asset and any other exogeneous economic variable is an issue of major importance both for practioners and for academics. The determination of extreme dependences is crucial for financial and for insurance institutions involved in risk management. It is also fundamental for the establishment of a rational investment policy striving for the best diversification of the various sources of risk. In all these situations, the objective is to prevent or at least minimize the simultaneous occurrence of large losses across the different positions held in the portfolio.

*We acknowledge helpful discussions and exchanges with J.P. Laurent and V. Pisarenko. This work was partially supported by the James S. Mc Donnell Foundation 21st century scientist award/studying complex system.

From an academic perspective, taking into account the extreme dependence properties provide useful yardsticks and important constraints for the construction of models, which should not underestimate or overestimate risks. From the point of view of univariate statistics, extreme values theory provides the mathematical framework for the classification and quantification of very large risks. It has been used for instance to study the phenomenon of contagion (as an example, see [Longin and Solnik (2001)] for a study of contagion across international equity markets). This has been made possible by the existence of a "universal" behavior summarized by the Gnedenko-Pickands-Balkema-de Haan theorem which gives a natural limit law for peak-over-threshold values in the form of the Generalized Pareto Distribution [Embrechts et al. (1997), pp 152-168]. In constrast, no such result is yet available in the multivariate case. In such absence of theoretical guidelines, the alternative is therefore to impose some dependence structure in a rather ad hoc and arbitrary way. This was the stance taken for instance in [Longin and Solnik (2001)].

This approach, where the dependence structure is not determined from empirical facts or from an economic model, is not fully satisfying. As a remedy, we propose a new approach, which does not directly rely on multivariate extreme values theory, but rather derives the extreme dependence structure from the characteristics of a financial model of assets. Specifically, we use the general class of factor models, which is probably one of the most versatile and relevant one, and whose introduction in finance can be traced back at least to [Ross (1976)]. The factor models are now widely used in many branches of finance, including stock return models, interest rate models [Vasicek (1977), Brennan and Schwarz (1978), Cox et al. (1985)], credit risks models [Carey (1998), Gorby (2000), Lucas et al. (2001)], etc., and are found at the core of many theories and equilibrium models.

Here, we will focus our efforts on the characterization of the extreme dependence between stock returns and the market return. The role of the market return as a factor explaining the evolution of individual stock returns is supported both by theoretical models such as the Capital Asset Pricing Model [Sharpe (1964), Lintner (1965), Mossin (1966)] or the Arbitrage Pricing Theory [Ross (1976)] and by empirical studies [Fama and Mc Beth (1973), Kandel and Staumbaugh (1987)] among many others. It has even been shown in [Roll (1988)] that in certain dramatic circumstances, such as the October 1987 stock-market crash, the (global) market was the sole relevant factor needed to explain the stock market movements and the propagation of the crash across countries. Thus, the choice of factor models is a very natural starting point for studying extreme dependences from a general point of view. The main gain is that, without imposing any *a priori* ad hoc dependences other than the definition of the factor model, we shall be able to derive the general properties of extreme dependence between an asset and one of its factor and to empirically determine these properties by a simple estimation of the factor model parameters.

The plan of our presentation is as follows. Section 1 defines the concepts needed for the characterization and quantification of extreme dependences. In particular, we recall the definition of the coefficient of tail dependence, which captures in a single number the properties of extreme dependence between two random variables: the tail dependence is defined as the probability for a given random variable to be large assuming that another random variable is large, at the same probability level. We shall also need some basic notions on dependences between random variables using the mathematical concept of copulas. In order to provide some perspective on the following results, this section also contains the expression of some classical exemples of tail dependence coefficients for specific multivariate distributions.

Section 2 states our main result in the form of a general theorem allowing the calculation of the coefficient of tail dependence for any factor model with arbitrary distribution functions of the factors and of the idiosyncratic noise. We find that the factor must have sufficiently "wild" fluctuations (to be made precise below) in order for the tail dependence not to vanish. For normal distributions of the factor, the tail dependence is identically zero, while for regularly varying distributions (power laws), the tail dependence is in general non-zero.

Section 3 is devoted to the empirical estimation of the coefficients of tail dependence between individual stock returns and the market return. The tests are performed for daily stock returns. The estimated coefficients of tail dependence are found in good agreement with the fraction of historically realized extreme events that occur simultaneously with any of the ten largest losses of the market factor (these ten largest losses were not used to calibrate the tail dependence coefficient). We also find some evidence for comonotonicity in the crash of Oct. 1987, suggesting that this event is an "outlier," providing additional support to a previous analysis of large and extreme drawdowns.

We summarize our results and conclude in section 4.

1 Intrinsic measure of casual and of extreme dependences

This section provides a brief informal summary of the mathematical concepts used in this paper to characterize the normal and extreme dependences between asset returns.

1.1 How to characterize uniquely the full dependence between two random variables?

The answer to this question is provided by the mathematical notion of "copulas," initially introduced by [Sklar (1959)] [1], which allows one to study the dependence of random variables independently of the behavior of their marginal distributions. Our presentation focuses on two variables but is easily extended to the case of N random variables, whatever N may be. Sklar's Theorem states that, given the joint distribution function $F(\cdot,\cdot)$ of two random variables X and Y with marginal distribution $F_X(\cdot)$ and $F_Y(\cdot)$ respectively, there exists a function $C(\cdot,\cdot)$ with range in $[0,1]\times[0,1]$ such that

$$F(x,y) = C(F_X(x), F_Y(y)) \ , \tag{1}$$

for all (x,y). This function C is the *copula* of the two random variables X and Y, and is unique if the random variables have continous marginal distributions. Moreover, the following result shows that copulas are intrinsic measures of dependence. If $g_1(X), g_2(Y)$ are strictly increasing on the ranges of X, Y, the random variables $\tilde{X} = g_1(X), \tilde{Y} = g_2(Y)$ have exactly the same copula C [Lindskog (1999)]. The copula is thus invariant under strictly increasing transformation of the variables. This provides a powerful way of studying scale-invariant measures of associations. It is also a natural starting point for construction of multivariate distributions.

1.2 Tail dependence between two random variables

A standard measure of dependence between two random variables is provided by the correlation coefficient. However, it suffers from at least three deficiencies. First, as stressed by [Embrechts et al. (1999)], the correlation coefficient is an adequate measure of dependence only for elliptical distributions and for events of moderate sizes. Second, the correlation coefficient measures only the degree of linear dependence and does not account of any other nonlinear functional dependence between the random variables. Third, it agregates both the marginal behavior of each random variable and their dependence. For instance, a simple change in the marginals implies in general a change in the correlation coefficient, while the copula and, therefore the dependence, remains unchanged. Mathematically speaking, the correlation coefficient is said to lack the property of invariance under increasing changes of variables.

[1] The reader is refered to [Joe (1997), Frees and Valdez (1998)] or [Nelsen (1998)] for a detailed survey of the notion of copulas and a mathematically rigorous description of their properties.

Since the copula is the unique and intrinsic measure of dependence, it is desirable to define measures of dependences which depend only on the copula. Such measures have in fact been known for a long time. Examples are provided by the concordance measures, among which the most famous are the Kendall's tau and the Spearman's rho (see [Nelsen (1998)] for a detailed exposition). In particular, the Spearman's rho quantifies the degres of functional dependence between two random variables: it equals one (minus one) when and only when the first variable is an increasing (decreasing) function of the second variable. However, as for the correlation coefficient, these concordance measures do not provide a useful measure of the dependence for extreme events, since they are constructed over the whole distributions.

Another natural idea, widely used in the contagion literature, is to work with the conditional correlation coefficient, conditioned only on the largest events. But, as stressed by [Boyer et al. (1997)], such conditional correlation coefficient suffers from a bias: even for a constant *unconditional* correlation coefficient, the *conditional* correlation coefficient changes with the conditioning set. Therefore, changes in the conditional correlation do not provide a characteristic signature of a change in the true correlations. The conditional concordance measures suffer from the same problem.

In view of these deficiencies, it is natural to come back to a fundamental definition of dependence through the use of probabilities. We thus study the conditional probality that the first variable is large conditioned on the second variable being large too: $\bar{F}(x|y) = \Pr\{X > x|Y > y\}$, when x and y goes to infinity. Since the convergence of $\bar{F}(x|y)$ may depend on the manner with which x and y go to infinity (the convergence is not uniform), we need to specify the path taken by the variables to reach the infinity. Recalling that it would be preferable to have a measure which is independent of the marginal distributions of X and Y, it is natural to reason in the quantile space. This leads to choose $x = {F_X}^{-1}(u)$ and $y = {F_Y}^{-1}(u)$ and replace the conditions $x, y \to \infty$ by $u \to 1$. Doing so, we define the so-called coefficient of upper tail dependence [Coles et al. (1999), Lindskog (1999), Embrechts et al. (2001)]:

$$\lambda_+ = \lim_{u \to 1^-} \Pr\{X > {F_X}^{-1}(u) \mid Y > {F_Y}^{-1}(u)\} \ . \tag{2}$$

As required, this measure of dependence is independent of the marginals, since it can be expressed in term of the copula of X and Y as

$$\lambda_+ = \lim_{u \to 1^-} \frac{1 - 2u + C(u,u)}{1-u} \ . \tag{3}$$

This representation shows that λ_+ is symmetric in X and Y, as it should for a reasonable measure of dependence.

In a similar way, we define the coefficient of lower tail dependence as the probabilty that X incurs a large loss assuming that Y incurs a large loss at the same probability level

$$\lambda_- = \lim_{u \to 0^+} \Pr\{X < {F_X}^{-1}(u) \mid Y < {F_Y}^{-1}(u)\} = \lim_{u \to 0^+} \frac{C(u,u)}{u} \ . \tag{4}$$

The values of the coefficients of tail dependence are known explicitely for a large number of different copulas. For instance, the Gaussian copula, which is the copula derived from de Gaussian multivariate distribution, has a zero coefficient of tail dependence. In contrast, the Gumbel's copula used by [Longin and Solnik (2001)] in the study of the contagion between international equity markets, which is defined by

$$C_\theta(u,v) = \exp\left(-\left[(-\ln u)^\theta + (-\ln v)^\theta\right]^{\frac{1}{\theta}}\right), \qquad \theta \in [0,1], \tag{5}$$

has an upper tail coefficient $\lambda_+ = 2 - 2^\theta$. For all θ's smaller than one, λ_+ is positive and the Gumbel's copula is said to present tail dependence, while for $\theta = 1$, the Gumbel copula is said to be asymptotically independent. One should however use this terminology with a grain of salt as "tail independence" (quantified by $\lambda_+ = 0$ or $\lambda_- = 0$) does not imply necessarily that large events occur independently (see [Coles et al. (1999)] for a precise discussion of this point).

2 Tail dependence of factor models

2.1 General result

We now state our main theoretical result. Let us consider two random variables X and Y of cumulative distribution functions $F_X(X)$ and $F_Y(Y)$, where X represents the return of a single stock and Y is the market return. Let us also introduce an idiosyncratic noise ε, which is assumed independent of the market return Y. The factor model is defined by the following relationship between the individual stock return X, the market return Y and the idiosyncratic noise ε:

$$X = \beta \cdot Y + \varepsilon \ . \tag{6}$$

β is the usual coefficient introduced by the Capital Asset Pricing Model [Sharpe (1964)]. Let us stress that ε may embody other factors Y', Y'', ..., as long as they remain independent of Y. Under such conditions and a few other technical assumptions detailed in the theorem established in appendix A, the coefficient of (upper) tail dependence between X and Y defined in (2) is obtained as

$$\lambda_+ = \int_{\max\left\{1, \frac{l}{\beta}\right\}}^{\infty} dx \ f(x) \ , \tag{7}$$

where l denotes the limit, when $u \rightarrow 1$, of the ratio ${F_X}^{-1}(u)/{F_Y}^{-1}(u)$, and $f(x)$ is the limit, when $t \rightarrow +\infty$, of $t \cdot P_Y(tx)/\bar{F}_Y(t)$. P_Y is the distribution density of Y and $\bar{F}_Y = 1 - F_Y$ is the complementary cumulative distribution function of Y. A similar expression obviously holds, *mutatis mutandis*, for the coefficient of lower tail dependence.

We now derive two direct consequences of this result (7) (see corollary 1 and 2 in appendix A), concerning rapidly varying and regularly varying factors.

2.2 Absence of tail dependence for rapidly varying factors

Let us assume that the factor Y and the idiosyncratic noise ε are normally distributed (the second assumption is made for simplicity and will be relaxed below). As a consequence, the joint distribution of (X, Y) is the bivariate Gaussian distribution. Refering to the results stated in section 1.2, we conclude that the copula of (X, Y) is the Gaussian copula whose coefficient of tail dependence is zero. In fact, it is easy to show that $\lambda = 0$ for any distribution of ε.

More generally, let us assume that the distribution of the factor Y is rapidly varying, which describes the Gaussian, exponential and any distribution decaying faster than any power-law. Then, the coefficient of tail dependence is identically zero. This result holds for any arbitrary distribution of the idiosyncratic noise (see corollary 1 in apendix B.1).

This statement is somewhat counter-intuitive since one could expect *a priori* that the coefficient of tail dependence does not vanish as soon as the tail of the distribution of factor returns is fatter than the tail the distribution noise returns. However, this example indicates that this is not the case and, in order to get a non-vanishing tail-dependence, the fluctuations of the factor must be 'wild' enough, which is not realized with rapidly varying distributions.

2.3 Coefficient of tail dependence for regularly varying factors

2.3.1 Example of the factor model with Student distribution

In order to account for the power-law tail behavior observed for the distributions of assets returns it is logical to consider that the factor and the indiosyncratic noise also have power-law tailed distributions. As an illustration, we will assume that Y and ε are distributed according to a Student's distribution with the same number of degrees of freedom ν (and thus same tail exponent ν). Let us denote by σ the scale factor of the distribution of ε while the scale factor of the distribution of Y is choosen equal to one[2]. Applying the theorem previously established, we find that $f(x) = \nu/x^{\nu+1}$ and $l = \beta\left[1+\left(\frac{\sigma}{\beta}\right)^{\nu}\right]^{1/\nu}$, so that the coefficient of tail dependence is

$$\lambda_{\pm} = \frac{1}{1+\left(\frac{\sigma}{\beta}\right)^{\nu}}, \qquad \text{and } \beta > 0. \tag{8}$$

As expected, the tail dependence increases as β increases and as σ decreases. The dependence with respect to ν is less intuitive. In particular, let ν go to infinity. Then, $\lambda \to 0$ if $\sigma > \beta$ and $\lambda \to 1$ for $\sigma < \beta$. This is surprising as one could argue that, as $\nu \to \infty$, the Student distribution tends to the Gaussian law. As a consequence, one would expect the same coefficient of dependence $\lambda_{\pm} = 0$ as for rapidly varying functions. The reason for the non-certain convergence of $\lambda_{\pm}$ to zero as $\nu \to \infty$ is rooted in a subtle non-commutativity (and non-uniform convergence) of the two limits $\nu \to \infty$ and $u \to 1$. Indeed, when taking first the limit $u \to 1$, the result $\lambda \to 1$ for $\beta > \sigma$ indicates that a sufficiently strong factor coefficient β always ensures the validity of the power law regime, whatever the value of ν. Correlatively, in this regime $\beta > \sigma$, $\lambda_{\pm}$ is an increasing function of ν.

2.4 General result

We now provide the general result valid for any regularly varying distribution. Let the factor Y follows a regularly varying distribution with tail index α: in other words, the complementary cumulative distribution of Y is such that $\bar{F}_Y(y) = L(y) \cdot y^{-\alpha}$, where $L(y)$ is a slowly varying function, i.e:

$$\lim_{t\to\infty} \frac{L(ty)}{L(t)} = 1, \qquad \forall y > 0. \tag{9}$$

Corollary 2 in appendix B.2 shows that

$$\lambda = \frac{1}{\left[\max\left\{1, \frac{l}{\beta}\right\}\right]^{\alpha}}, \tag{10}$$

where l denotes the limit, when $u \to 1$, of the ratio ${F_X}^{-1}(u)/{F_Y}^{-1}(u)$. In the case of particular interest when the distribution of ε is also regularly varying with tail index α and if, in addition, we have $\bar{F}_Y(y) \sim C_y \cdot y^{-\alpha}$ and $\bar{F}_\varepsilon(\varepsilon) \sim C_\varepsilon \cdot \varepsilon^{-\alpha}$, for large y and ε, then the coefficient of tail dependence is a simple function of the ratio C_ε/C_y of the scale factors:

$$\lambda = \frac{1}{1+\beta^{-\alpha}\cdot\frac{C_\varepsilon}{C_y}}. \tag{11}$$

When the tail indexes α_Y and α_ε of the distribution of the factor and the residue are different, then $\lambda = 0$ for $\alpha_Y < \alpha_\varepsilon$ and $\lambda = 1$ for $\alpha_Y > \alpha_\varepsilon$.

[2]Such a choice is always possible via a rescaling of the coefficient β.

Now that we have entirely characterized the tail dependence for the factor model, we will use these results to estimate empirically the tail dependence between different stock returns and the market return and test our prediction on historical events.

3 Empirical study

We now apply our theoretical results to the daily returns of a set of stocks traded on the New York Stock Exchange. In order to estimate the parameters of the factor model (6), the Standard and Poor's 500 index, which represents about 80% of the total market capitalization, is choosen to represent the common "market factor."

We describe the set of selected stocks in the next sub-section. Next, we estimate the parameter β in (6) and check the independence of the market returns and the residues. Then, applying the commonly used hypothesis according to which the tail of the distribution of assets return is a power law (see [Longin (1996), Lux (1996), Pagan (1996), Gopikrishnan et al. (1998)]), we estimate the tail index and the scale factor of these distributions, which allows us to calculate the coefficients of tail dependence between each asset return and the market return. Finally, we perform an analysis of the historical data to check the compatibility of our prediction on the fraction of realized large losses of the assets that occur simultaneously with the large losses of the market.

The results of our analysis are reported below in terms of the returns rather than in terms of the excess returns above the risk free interest rate, in apparent contradiction with the prescription of the CAPM. However, for daily returns, the difference between returns and excess returns is negligible. Indeed, we checked that neglecting the difference between the returns and the excess returns does not affect our results by re-running all the study described below in terms of the excess returns and found that the tail dependence did not change by more than 0.1%.

3.1 Description of the data

We study a set of twenty assets traded on the New York Stock Exchange. The criteria presiding over the selection of the assets (see column 1 of table 1) are that (1) they are among the stocks with the largest capitalizations, but (2) each of them should have a weight smaller than 1% in the Standard and Poor's 500 index, so that the dependence studied here does not stem trivially from their overlap with the market factor (taken as the Standard and Poor's 500 index).

The time interval we have considered ranges from July 03, 1962 to December 29, 2000, corresponding to 9694 data points, and represents the largest set of daily data available from the Center for Research in Security Prices (CRSP) . This large time interval is important to let us collect as many large fluctuations of the returns as is possible in order to sample the extreme tail dependence. Moreover, in order to allow for a non-stationarity over the four decades of the study, to check the stability of our results and to test the stationnarity of the tail dependence over the time, we split this set into two subsets. The first one ranges from July 1962 to December 1979, a period with few very large return amplitudes, while the second one ranges from January 1980 to December 2000, a period which witnessed several very large price changes (see table 1) which shows the good stability of the standard deviation between the two sub-periods while the higher cumulants such as the excess kurtosis often increased dramatically in the second sub-period for most assets). The table 1 presents the main statistical properties of our set of stocks during the three time intervals. All assets exhibit an excess kurtosis significantly different from zero over the three time interval, which is inconsistent with the assumption of Gaussianly distributed returns. While the standard deviations

remain stable over time, the excess kurtosis increases significantly from the first to the second period. This is in resonance with the financial community's belief that stock price volatility has increased over time, a still controversial result [Jones and Wilson, 1989].

3.2 Calibration of the factor model

The determination of the parameters β and of the residues ε entering in the definition of the factor model (6) is performed for each asset by regressing the stocks returns on the market return. The coefficient β is thus given by the ordinary least square estimator, which is consistent as long as the residues are white noise and with zero mean and finite variance. The idiosyncratic noise ε is obtained by substracting β times the market return to the stock return. Table 2 presents the results for the three periods we consider. For each period, we give the value of the estimated coefficient β and the correlation coefficient between the market returns and the idiosyncratic noise. A Fisher's test is shows that, at the 95% confidence level, none af these correlation coefficients is significantly different from zero. This does not necessarily ensures the independence of the idiosyncratic noises with respect to the market return, but is nonetheless a positive result for the validity of the factor decomposition (6).

The coefficient β's we obtain by regressing each asset returns on the Standard & Poor's 500 returns are very close to within their uncertainties to the β's given by the CRSP database, which are estimated by regressing the assets returns on the value-weighted market portfolio. Thus, the choice of the Standard and Poor's 500 index to represent the whole market portfolio is reasonable.

3.3 Estimation of the tail indexes

Assuming that the distributions of stocks and market returns are asymptotically power laws [Longin (1996), Lux (1996), Pagan (1996), Gopikrishnan et al. (1998)], we now estimate the tail index of the distribution of each stock and their corresponding residue by the factor model, both for the positive and negative tails. Each tail index α is given by Hill's etimator:

$$\hat{\alpha} = \left[\frac{1}{k} \sum_{j=1}^{k} \log x_{j,N} - \log x_{k,N} \right]^{-1} , \tag{12}$$

where $x_{1,N} \geq x_{2,N} \geq \cdots \geq x_{N,N}$ denotes the ordered statistics of the sample containing N independent and identically distributed realizations of the variable X.

Hill's estimator is asymptotically normally distributed with mean α and variance α^2/k. But, for finite k, it is known that the estimator is biased. As the range k increases, the variance of the estimator decreases while its bias increases. The competition between these two effects implies that there is an optimal choice for $k = k^*$ which minimizes the mean squared error of the estimator. To select this value k^*, one can apply the [Danielsson and de Vries (1997)]'s algorithm which is an improvement over the [Hall (1990)]'s subsample bootstrap procedure. One can also prefer the more recent [Danielsson et al. (2001)]'s algorithm for the sake of parsimony. We have tested all three algorithms to determine the optimal k^*. It turns out that the [Danielsson et al. (2001)]'s algorithm developped for high frequency data is not well adapted to samples containing less than 100,000 data points, as is the case here. Thus, we have focused on the two other algorithms. An accurate determination of k^* is rather difficult with any of them, but in every case, we found that the relevant range for the tail index estimation was between the 1% and 5% quantiles. Tables 3 and 4 give the estimated tail index for each asset and residues at the 1%, 2.5% and 5% quantile, for both the positive and the negative tails for the two time sub-intervals. The second time interval from January

1980 to December 2000 is characterized by values of the tail indexes that are homogeneous over the various quantiles and range between 3 and 4 for the negative tails and between 3 and 5 for the positive tails. There is slightly more dispersions in the first time interval from July 1962 to December 1979.

For each asset and their residue of the regression on the market factor, we tested whether the hypothesis, according to which the tail index measured for each asset and each residue is the same as the tail index of the Standard & Poor's 500 index, can be rejected at the 95% confidence level, for a given quantile. The values which reject this hypothesis are indicated by a star in the tables 3 and 4. During the second time interval from January 1980 to December 2000, only four residues have a tail index significantly different from that of that Standard & Poor's 500, and only in the negative tail. The situation is not as good during the first time interval, especially for the negative tail, for which not less 13 assets and 10 residues out of 20 have a tail index significantly different from the Standard & Poor's 500 ones, for the 5% quantile.

To summarize, our tests confirm that the tail indexes of most stock return distributions range between three and four, even though no better precision can be given with good significance. Moreover, in most cases, we can assume that both the asset, the factor and the residue have the same tail index. We can also add that, as asserted by [Loretan and Phillips (1994)] or [Longin (1996)], we cannot reject the hypothesis that the tail index remains the same over time. Nevertheless, it seems that during the first period from July 1962 to December 1979, the tail indexes were sightly larger than during the second period from January 1980 to december 2000.

3.4 Determination of the coefficient of tail dependence

Using the just established empirical fact that we cannot reject the hypothesis that the assets, the market and the residues have the same tail index, we can use the theorem of Appendix A and its second corollary stated in section 2. This leads us to conclude that one cannot reject the hypothesis of a non-vanishing tail dependence between the assets and the market. In addition, the coefficient of tail dependence is given by equation (11). In order to determine its value, we need to estimate the scale factors for the different assets to derive their coefficient of tail dependence, according to the formula (11).

We proceed as follows. Consider a variable X which asymptotically follows a power law distribution $\Pr\{X > x\} \sim C \cdot x^{-\alpha}$. Given a rank ordered sample $x_{1,N} \geq x_{2,N} \geq \cdots \geq x_{N,N}$, the scale factor C can be consistently estimated from the k largest realizations by

$$\hat{C} = \frac{k}{N} \cdot (x_{k,N})^{\alpha} \, . \tag{13}$$

In the tail, this estimator is independent of k. Thus, denoting by $\hat{C}_Y$ and $\hat{C}_\varepsilon$ the scale factors of the factor Y and of the noise ε defined in equation (6), the estimator of the coefficient of tail dependence is

$$\hat{\lambda} = \frac{1}{1 + \hat{\beta}^{-\alpha} \cdot \frac{\hat{C}_Y}{\hat{C}_\varepsilon}} = \frac{1}{1 + \left(\frac{\varepsilon_{k,N}}{\hat{\beta} \cdot y_{k,N}}\right)^{\alpha}} \, . \tag{14}$$

Since the tail indices α are impossible to determine with sufficient accuracy other than saying that the α probably fall in the interval $3 - 4$ as we have seen above, our strategy is to determine $\hat{\lambda}$ using (14) for three different common values $\alpha = 3$, 3.5 and 4. This procedure allows us to test for the sensitivity of the scale factor and therefore of the tail coefficient with respect to the uncertain value of the tail index.

Table 5 gives the values of the coefficients of lower tail dependence over the whole time interval from July 1962 to December 2000, under the assumption that the tail index α equals 3. The coefficient of tail dependence is estimated over the first centile, the first quintile and the first decile to also test for any possible

sensitivity on the tail asymptotics. For each of these quantiles, the mean values, their standard deviations and their minimum and maximum values are given. We first remark that the standard deviation of the tail dependence coefficient remains small compared with its average value and that the minimum and maximum values cluster closely around its mean value. This shows that the coefficient of tail dependence is well-estimated by its mean over a given quantile. Secondly, we find that these estimated coefficients of tail dependence exhibit a good stability over the varous quantiles. These two observations enable us to conclude that the average coefficient of tail dependence over the first centile is sufficient to provide a good estimate of the true coefficient of tail dependence.

Tables 6, 7 and 8 summarize the different values of the coefficient of tail dependence for both the positive and the negative tails, under the assumptions that the tail index α equals 3, 3.5 and 4 respectively, over the three considered time intervals. Overall, we find that the coefficients of tail dependence are almost equal for both the negative and the positive tail and that they are not very sensitive to the value of the tail index in the interval considered. More precisely, during the first time interval from July 1962 to December 1979 (table 6), the tail dependence is symetric in both the upper and the lower tail. During the second time interval from January 1980 to December 2000 and over the whole time interval (tables 7 and 8), the coefficient of lower tail dependence is slightly but systematically larger than the upper one. Moreover, since these coefficients of tail dependence are all less than $1/2$, they decrease when the tail index α increases and the smaller the coefficient of tail dependence, the larger the decay.

During the first time interval, most of the coefficients of tail dependence range between 0.15 and 0.35 in both tails, while during the second time interval, almost all range between 0.10 and 0.25 in the lower tail and between 0.10 and 0.20 in the upper one. Thus, the tail dependence is smaller during the last period than during the first one. This result is interesting because it associates the smaller (respectively larger) tail dependence to the second (resp. first) period of larger (resp. smaller) volatility, as quantified for instance by the excess kurtosis.

3.5 Comparison with the historical extremes

Our determination of the coefficient of tail dependences provides predictions on the probability that future large moves of stocks may be simultaneous to large moves of the market. This begs for a check over the available historical period to determine whether our estimated coefficients of tail dependence are compatible with the realized historical extremes.

For this, we consider the ten largest losses of the Standard & Poor's 500 index during the two time sub-intervals[3]. Since λ_- is by definition equal to the probability that a given asset incurs a large loss (say, one of its ten largest losses) conditional on the occurrence of one of the ten largest losses of the Standard & Poor's 500 index, the probability, for this asset, to undergo n of its ten largest losses simultaneously with any of the ten largest losses of the Standard & Poor's 500 index is given by the binomial law with parameter λ_-:

$$P_{\lambda_-}(n) = \binom{10}{n} \lambda_-{}^n \, (1-\lambda_-)^{(10-n)}. \tag{15}$$

We stress that our consideration of only the ten largest drops ensures that the present test is not embodied in the determination of the tail dependence coefficient, which has been determined on a robust procedure over the 1%, 5% and 10% quantiles. We checked that removing these then largest drops does not modify the determination of λ_-. Our present test can thus be considered as "out-of-sample," in this sense.

[3]We do not consider the whole time interval since the ten largest losses over the whole period coincide with the ten largest ones over the second time subinterval, which would bias the statistics towards the second time interval.

Table 9 presents, for the two time sub-intervals, the number of extreme losses among the ten largest losses incured by a given asset which occured simultaneously with one of the ten largest losses of the standard & Poor's 500 index. For each asset, we give the probability of occurence of such a realisation, according to (15). We notice that during the first time interval, only two assets are incompatible, at the 95% confidence level, with the value of λ_- previously determined: Du Pont (E.I.) de Nemours & Co. and Texaco Inc. In constrast, during the second time interval, four assets reject the value of λ_-: Coca Cola Corp., Pepsico Inc., Pharmicia Corp. and Texas Instruments Inc.

These results are very encouraging. However, there is a noticeable systematic bias. Indeed, during the first time interval, 17 out of the 20 assets have a realized number of large losses lower than their expected number (according to the estimated λ_-), while during the second time interval, 19 out of the 20 assets have a realized number of large losses larger than their expected one. Thus, it seems that during the first time interval the number of large losses is overestimated by λ_- while it is underestimated during the second time interval.

We propose to explain the underestimation of the number of large losses between January 1980 and December 2000 by a possible comonitonicity that occurred during the October 1987 crash. Indeed, on October 19, 1987, 12 out of the 20 considered assets incurred their most severe loss, which strongly suggests a comonotonic effect. Table 10 shows the same results as in table 9 but corrected by substracting this comonotonic effect to the number of large losses. The compatibility between the number of large losses and the estimated λ_- becomes significantly better since only Pepsico Inc. and Pharmicia Corp. are still rejected, and only 16 assets out of 20 are underestimated, representing a significant decrease of the bias.

Previous works have shown that, in period of crashes, the market conditions change, herding effects may become more important and almost dominant, so that the market enters an unusual regime, which can be characterized by outliers present in the distribution of drawdowns [Johansen and Sornette (2002)]. Our detection of an anomalous comonotonicity can thus be considered as an independent confirmation of the existence of this abnormal regime.

Obviously, the overestimation of the number of large losses during the first time interval can not be ascribe to the comonotonicity of very large events, which in fact only occurred once for the Coca-Cola Corp. This overestimation is probably linked with the low "volatility" of the market during this period, which can have two effects. The first one is to lead to a less accurate estimation of the scale factor of the power-law distribution of the assets. The second one is that a market with smaller volatility produices fewer large losses. As a consequence, the asymptotic regime for which the relation $\Pr\{X < {F_X}^{-1}(u) | Y < {F_Y}^{-1}(u)\} \simeq \lambda_-$ holds may not be reached in the sample, and the number of recorded large losses remain lower than that asymptotically expected.

4 Concluding remarks

We have used the framework offered by factor models in order to derive a general theoretical expression for the coefficient of tail dependence between a random variable and any of its explanatory factor. The coefficient of tail dependence represents the probability that an asset incurs a large loss (say), assuming that the market has also undergone a large loss. We find that factors characterized by rapidly varying distributions, such as Normal or exponential distributions, always lead to a vanishing coefficient of tail dependence with other stocks. In constrast, factors with regularly varying distributions, such as power-law distributions, can exhibit tail dependence with other stocks, provided that the idiosyncratic noise distributions of the corresponding stocks are not fatter-tailed than the factor.

Applying this general result to individual daily stock returns, we have been able to estimate the coefficient of tail dependence between the returns of each stock and those of the market. This determination of the

tail dependence relies only on the simple estimation of the parameters of the underlying factor model and on the tail parameters of the distribution of the factor and of the idiosyncratic noise of each stock. As a consequence, the two strong advantages of our approach are the following.

- The coefficients of tail dependence are estimated non-parametrically. Indeed, we never specify any explicit expression of the dependence structure, contrary to most previous works (see [Longin and Solnik (2001), Malevergne and Sornette (2001)] or [Patton (2001)] for instance);

- Our theoretical result enables us to estimate an extreme parameter, not accessible by a direct statistical inference. This is achieved by the measurement of parameters whose estimation involves a significant part of the data with sufficient statistics.

Having performed this estimation, we have checked the comptatibility of these estimated coefficients of tail dependence with the historically realized extreme losses observed in the empirical time series. A good agreement is found, notwithstanding a slight bias which leads to an overestimate of the occurence of large events during the period from July 1962 to December 1979 and to an underestimate during the time interval from January 1980 to December 2000.

This bias can be explained by the low volatility of the market during the first period and by a comonotonicity effect, due to the October 1987 crach, during the second period. Indeed, from july 1962 to December 1979, the volatility was so low that the distributions of returns have probably not sampled their tails sufficiently far the probability of large conditional losses to be represented by its asymptotic expression given by the coefficient of tail dependence. The situation is very different for the period from january 1980 to December 2000. On October 19, 1987, many assets incurred their largest loss ever. This is presumably the manifestation of an 'abnormal' regime probably due to herding effects and irrational behaviors and has been previously characterized as yielding signatures in the form of outliers in the distribution of drawdowns.

Finally, the observed lack of stationarity exhibited by the coefficient of tail dependence across the two time sub-intervals suggests the importance of going beyond a stationary view of tail dependence and of studying its dynamics. This question, which could be of great interest in the context of the contagion problem, is left for a future work.

Our study has focused on the dependence between different risks. In fact, our theorem can obviously be applied to extreme temporal dependences, when the variable follows an autoregressive process. This should provide an estimate of the probability that a large loss (respectively gain) is followed by another large loss (resp. gain) in the following period. Such information is very interesting in investment and hedging strategies.

A Proof of the theorem

A.1 Statement

We consider two random variables X and Y, related by the relation

$$X = \beta \cdot Y + \varepsilon, \tag{16}$$

where ε is a random variable independent of Y and β a non-random positive coefficient.

Let P_Y and F_Y denote respectively the density with respect to the Lebesgue measure and the distribution function of the variable Y. Let F_X denotes the distribution function of X and F_ε the distribution function of ε. We state the following theorem:

THEOREM 1
Assuming that

- *H0: The variables Y and ε have distribution functions with infinite support,*
- *H1: For all $x \in [1, \infty)$,*
$$\lim_{t \to \infty} \frac{t\ P_Y(tx)}{\bar{F}_Y(t)} = f(x), \tag{17}$$
- *H2: There are real numbers $t_0 > 0$, $\delta > 0$ and $A > 0$, such that for all $t \geq t_0$ and all $x \geq 1$*
$$\frac{\bar{F}_Y(tx)}{\bar{F}_Y(t)} \leq \frac{A}{x^\delta}, \tag{18}$$
- *H3: There is a constant $l \in \mathbb{R}_+$, such that*
$$\lim_{u \to 1} \frac{{F_X}^{-1}(u)}{{F_Y}^{-1}(u)} = l, \tag{19}$$

then, the coefficient of (upper) tail dependence of (X, Y) is given by

$$\lambda = \int_{\max\{1, \frac{l}{\beta}\}}^{\infty} dx\ f(x). \tag{20}$$

A.2 Proof

We first give a general expression for the probability for X to be larger than $F_X^{-1}(u)$ knowing that Y is larger than $F_Y^{-1}(u)$:

LEMMA 1
The probability that X is larger than $F_X^{-1}(u)$ knowing that Y is larger than $F_Y^{-1}(u)$ is given by :

$$\Pr\left[X > F_X^{-1}(u) | Y > F_Y^{-1}(u)\right] = \frac{F_Y^{-1}(u)}{1-u} \int_1^\infty dx\ P_Y\left(F_Y^{-1}(u)\ x\right) \cdot \bar{F}_\varepsilon\left[{F_X}^{-1}(u) - \beta F_Y^{-1}(u)\ x\right] . \tag{21}$$

Proof :

$$
\begin{aligned}
\Pr\{X > F_X{}^{-1}(u), Y > F_Y{}^{-1}(u)\} &= \mathrm{E}\left[1_{\{X>F_X{}^{-1}(u)\}} \cdot 1_{\{Y>F_Y{}^{-1}(u)\}}\right] && (22)\\
&= \mathrm{E}\left[\mathrm{E}\left[1_{\{X>F_X{}^{-1}(u)\}} \cdot 1_{\{Y>F_Y{}^{-1}(u)\}}|Y\right]\right] && (23)\\
&= \mathrm{E}\left[1_{\{Y>F_Y{}^{-1}(u)\}} \cdot \mathrm{E}\left[1_{\{X>F_X{}^{-1}(u)\}}|Y\right]\right] && (24)\\
&= \mathrm{E}\left[1_{\{Y>F_Y{}^{-1}(u)\}} \cdot \mathrm{E}\left[1_{\{\varepsilon>F_X{}^{-1}(u)-\beta Y\}}\right]\right] && (25)\\
&= \mathrm{E}\left[1_{\{Y>F_Y{}^{-1}(u)\}} \cdot \bar{F}_\varepsilon(F_X{}^{-1}(u) - \beta Y)\right] && (26)
\end{aligned}
$$

Assuming that the variable Y admits a density P_Y with respect to the Lebesgue measure, this yields

$$
\Pr\{X > F_X{}^{-1}(u), Y > F_Y{}^{-1}(u)\} = \int_{F_Y^{-1}(u)}^{\infty} dy\; P_Y(y) \cdot \bar{F}_\varepsilon[F_X{}^{-1}(u) - \beta y]\,. \tag{27}
$$

Performing the change of variable $x = F_Y{}^{-1}(u) \cdot y$, in the equation above, we obtain

$$
\Pr\{X > F_X{}^{-1}(u), Y > F_Y{}^{-1}(u)\} = F_Y^{-1}(u) \int_1^{\infty} dx\; P_Y(F_Y^{-1}(u)\,x) \cdot \bar{F}_\varepsilon[F_X{}^{-1}(u) - \beta F_Y^{-1}(u)\,x]\,, \tag{28}
$$

and, dividing by $\bar{F_Y}\left(F_Y{}^{-1}(u)\right) = 1 - u$, this concludes the proof. $\square$

Let us now define the function

$$
f_u(x) = \frac{F_Y^{-1}(u)}{1-u}\, P_Y(F_Y^{-1}(u)\,x) \cdot \bar{F}_\varepsilon[F_X{}^{-1}(u) - \beta F_Y^{-1}(u)\,x]\,. \tag{29}
$$

We can state the following result

LEMMA 2
Under assumption $H1$ and $H3$, for all $x \in [1, \infty)$,

$$
f_u(x) \longrightarrow 1_{\left\{x > \frac{l}{\beta}\right\}} \cdot f(x), \tag{30}
$$

almost everywhere, as u goes to 1.

Proof: Let us apply the assumption $H1$. We have

$$
\begin{aligned}
\lim_{u\to 1} \frac{F_Y^{-1}(u)}{1-u}\, P_Y(F_Y^{-1}(u)\,x) &= \lim_{t\to\infty} \frac{t\,P_Y(t\,x)}{\bar{F}_Y(t)}\,, && (31)\\
&= f(x). && (32)
\end{aligned}
$$

Applying now the assumption $H3$, we have

$$
\begin{aligned}
\lim_{u\to 1} F_X{}^{-1}(u) - \beta F_Y^{-1}(u)\,x &= \lim_{u\to 1} \beta F_Y^{-1}(u) \left(\frac{F_X{}^{-1}(u)}{\beta F_Y^{-1}(u)} - x\right) && (33)\\
&= \begin{cases} -\infty & \text{if } x > \frac{l}{\beta}, \\ \infty & \text{if } x < \frac{l}{\beta}, \end{cases} && (34)\\
& && (35)
\end{aligned}
$$

which gives
$$\lim_{u\to 1} \bar{F}_\varepsilon[{F_X}^{-1}(u) - \beta F_Y^{-1}(u)\ x] = 1_{\left\{x > \frac{l}{\beta}\right\}}, \tag{36}$$
and finally
$$\lim_{u\to 1} f_u(x) = \lim_{u\to 1} \frac{F_Y^{-1}(u)}{1-u}\, P_Y(F_Y^{-1}(u)\ x) \cdot \lim_{u\to 1} \bar{F}_\varepsilon[{F_X}^{-1}(u) - \beta F_Y^{-1}(u)\ x], \tag{37}$$
$$= 1_{\left\{x > \frac{l}{\beta}\right\}} \cdot f(x), \tag{38}$$
which concludes the proof. □

Let us now proove that there exists an integrable function $g(x)$ such that, for all $t \geq t_0$ and all $x \geq 1$, we have $f_t(x) \leq g(x)$. Indeed, let us write
$$\frac{t\ P_Y(tx)}{\bar{F}_Y(t)} = \frac{t\ P_Y(tx)}{\bar{F}_Y(tx)} \cdot \frac{\bar{F}_Y(tx)}{\bar{F}_Y(t)}\ . \tag{39}$$
For the leftmost factor in the right-hand-side of equation (39), we easily obtain
$$\forall t,\ \forall x \geq 1, \quad \frac{t\ P_Y(tx)}{\bar{F}_Y(tx)} \leq \frac{x^*\ P_Y(x^*)}{\bar{F}_Y(x^*)} \cdot \frac{1}{x}, \tag{40}$$
where x^* denotes the point where the function $\frac{x\ P_Y(x)}{\bar{F}_Y(x)}$ reaches its maximum. The rightmost factor in the right-hand-side of (39) is smaller than A/x^δ by assumption $H2$, so that
$$\forall t \geq t_0,\ \forall x \geq 1, \quad \frac{t\ P_Y(tx)}{\bar{F}_Y(t)} \leq \frac{x^*\ P_Y(x^*)}{\bar{F}_Y(x^*)} \cdot \frac{A}{x^{1+\delta}}\ . \tag{41}$$
Posing
$$g(x) = \frac{x^*\ P_Y(x^*)}{\bar{F}_Y(x^*)} \cdot \frac{A}{x^{1+\delta}}\ , \tag{42}$$
and recalling that, for all $\varepsilon \in \mathbb{R}$, $\bar{F}_\varepsilon(\varepsilon) \leq 1$, we have found an integrable function such that for some $u_0 \geq 0$, we have
$$\forall u \in [u_0, 1),\ \forall x \geq 1, \quad f_u(x) \leq g(x)\ . \tag{43}$$
Thus, applying Lebesgue's theorem of dominated convergence, we can assert that
$$\lim_{u\to 1} \int_1^\infty dx\ f_u(x) = \int_1^\infty dx\ 1_{\left\{x > \frac{l}{\beta}\right\}} \cdot f(x). \tag{44}$$
Since
$$\lim_{u\to 1} \int_1^\infty dx\ f_u(x) = \lim_{u\to 1} \Pr\left[X > F_X^{-1}(u) | Y > F_Y^{-1}(u)\right], \tag{45}$$
$$= \lambda, \tag{46}$$
the proof of theorem 1 is concluded. □

B Proofs of the corollaries

B.1 First corollary

COROLLARY 1
If the random variable Y has a rapidly varying distribution function, then $\lambda = 0$.

Proof : Let us write

$$\frac{t\ P_Y(tx)}{\bar{F}_Y(t)} = \frac{t\ P_Y(tx)}{\bar{F}_Y(tx)} \cdot \frac{\bar{F}_Y(tx)}{\bar{F}_Y(t)}. \tag{47}$$

For a rapidly varying function $\bar{F}_Y$, we have

$$\forall x > 1, \quad \lim_{t\to\infty} \frac{\bar{F}_Y(tx)}{\bar{F}_Y(t)} = 0, \tag{48}$$

while the leftmost factor of the right-hand-side of equation (47) remains bounded as t goes to infinity, so that

$$\lim_{t\to\infty} \frac{t\ P_Y(tx)}{\bar{F}_Y(tx)} \cdot \frac{\bar{F}_Y(tx)}{\bar{F}_Y(t)} = f(x) = 0\ . \tag{49}$$

Since $f(x) = 0$, we can apply lemma 2 without the hypothesis $H3$, which concludes the proof. □

B.2 Second corollary

COROLLARY 2
Let Y be regularly varying with index $(-\alpha)$, and assume that hypothesis $H3$ is satisfied. Then, the coefficient of (upper) tail dependence is

$$\lambda = \frac{1}{\left[\max\left\{1, \frac{l}{\beta}\right\}\right]^{\alpha}}, \tag{50}$$

where l denotes the limit, when $u \to 1$, of the ratio ${F_X}^{-1}(u)/{F_Y}^{-1}(u)$.

Proof : Karamata's theorem (see [Embrechts et al. (1997), p 567]) ensures that $H1$ is satisfied with $f(x) = \frac{\alpha}{x^{\alpha+1}}$, which is sufficient to proove the corollary. To go one step further, let us define

$$\bar{F}_y(y) = y^{-\alpha} \cdot L_1(y), \tag{51}$$

$$\bar{F}_\varepsilon(\varepsilon) = \varepsilon^{-\alpha} \cdot L_2(\varepsilon), \tag{52}$$

where $L_1(\cdot)$ and $L_2(\cdot)$ are slowly varying functions.

Using the proposition stated in [Feller (1971), p 278], we obtain, for the distribution of the variable X

$$\bar{F}_X(x) \sim x^{-\alpha} \left(\beta^\alpha \cdot L_1\left(\frac{x}{\beta}\right) + L_2(x)\right), \tag{53}$$

for large x.

Assuming now, for simplicity, that L_1 (resp. L_2) goes to a constant C_1 (resp. C_2), this implies that $H3$ is satistified, since

$$l = \lim_{u\to 1} \frac{{F_X}^{-1}(u)}{{F_Y}^{-1}(u)} = \beta\left[1 + \frac{C_2}{\beta^\alpha\ C_1}\right]^{\frac{1}{\alpha}}\ . \tag{54}$$

This allows us to obtain the equations (8) and (11). □

References

[Boyer et al. (1997)] Boyer, B.H., M.S Gibson and M. Lauretan, 1997, Pitfalls in tests for changes in correlations, International Finance Discussion Paper 597, Board of the Governors of the Federal Reserve System.

[Brennan and Schwarz (1978)] Brennan, M.J. and E.J. Schwarz, 1978, A continous time approach to the pricing of bonds, Journal of Banking and Finance, 3, 133-155.

[Carey (1998)] Carey, M., 1998, Credit risk in private debt portfolio, Journal of Finance, 53,56-61.

[Coles et al. (1999)] Coles, S., J. Heffernan and J. Tawn, 1999 Dependence measures for extreme value analyses, Extremes, 2, 339-365.

[Cox et al. (1985)] Cox, J.C., J.E. Ingersoll and S.A. Ross, 1985, A theory of the term structure of interest rates, Econometrica, 51, 385-408.

[Danielsson and de Vries (1997)] Danielsson, J. and C.G. de Vries, 1997, Tail index and quantile estimation with very high frequency data, Journal of Empirical Finance, 4, 241-257.

[Danielsson et al. (2001)] Danielsson, J., de Hann, L., Peng, L. and C.G. de Vries, 2001, Using a bootstrap method to choose the optimal sample fraction in tail index estimation, Journal of Multivariate analysis, 76, 226-248.

[Embrechts et al. (1997)] Embrechts, P., C.P. Kluppelberg, and T. Mikosh, 1997, Modelling Extremal Events (Springer-Verlag, Berlin).

[Embrechts et al. (1999)] Embrechts P., A.J. McNeil and D. Straumann, 1999, Correlation : Pitfalls and Alternatives. Risk, 69-71.

[Embrechts et al. (2001)] Embrechts P., A.J. McNeil and D. Straumann, 2001, Correlation and Dependency in Risk Management : Properties and Pitfalls, in : Dempster, M., ed., Value at Risk and Beyond (Cambridge University Press).

[Fama and Mc Beth (1973)] Fama, E. and J. Mc Beth, 1973, Risk, Return and equilibrium: empirical tests, Journal of Political Economy, 81, 607-636.

[Feller (1971)] Feller, W., 1971, An introduction to probability theory and its applications II (Wiley, New-York).

[Frees and Valdez (1998)] Frees, E. and E. Valdez, 1998, Understanding Relationships using copulas, North Americam Actuarial Journal 2, 1-25.

[Gopikrishnan et al. (1998)] Gopikrishnan, P., Meyer, M., Amaral, L.A.N. and Stanley, H.E., 1998, Inverse Cubic Law for the Distribution of Stock Price Variations, European Physical Journal B 3, 139 - 140.

[Gorby (2000)] Gorby, M.B., 2000, A comparative anatomy of credit risk models, Journal of Banking and Finance, 24,119-149.

[Hall (1990)] Hall,P., 1990, Using the bootstrap method to estimate mean squared error and select smoothing parameter in non parametric problems, Journal of Multivariate analysis, 32, 177-203.

[Joe (1997)] Joe, H., 1997, Multivariate models and dependence concepts (Chapman & Hall, London)

[Johansen and Sornette (2002)] Johansen, A. and D. Sornette, Large price drawdowns are outliers, forthcoming in Journal of Risk vol. 4(2), 2002.

[Jones and Wilson, 1989] Jones, C.P. and Wilson, J.W., Is Stock Price Volatility Increasing? Financial Analysts Journal 45, 20-26, 1989.

[Kandel and Staumbaugh (1987)] Kandel, S. and R. Staumbaugh, 1987, On correlations and the sensitivity of inference about mean-variance efficiency, journal of Financial Economics, 18, 61-90.

[Lindskog (1999)] Lindskog, F., Modelling Dependence with Copulas, $http://www.risklab.ch/Papers.html\#MTLindskog$

[Lintner (1965)] Lintner, J., 1965, The valuation of risk assets and the selection of risky investments in stock portfolios and the capital bugets, Review of Economics and Statitics, 41, 13-37.

[Longin (1996)] Longin, F.M., 1996, The asymptotic distribution of extrem stock market returns, Journal of Business, 96, 383-408.

[Longin and Solnik (2001)] Longin F. and B. Solnik, 2001, Extreme Correlation of International Equity Markets, Journal of Finance 56, 649-676.

[Loretan and Phillips (1994)] Loretan, M. and P.C.B. Phillips, 1994, Testing the covariance stationarity of heavy-tailed times series, Journal of Empirical Finance, 1, 211-248.

[Lucas et al. (2001)] Lucas A., P. Klaassen, P. Spreij and S. Straetmans, 2001, An analytic approach to credit risk of large corporate bond and loan portfolios, Journal of Banking and Finance, 25, 1635-1664.

[Lux (1996)] Lux, L., 1996, The stable Paretian hypothesis and the frequency of large returns: an examination of major German stocks, Applied Financial Economics 6, 463-475.

[Malevergne and Sornette (2001)] Malevergne, Y. and D. Sornette, 2001, Testing the Gaussian copula hypothesis for financial assets dependences, Working paper. (e-print available at http://papers.ssrn.com/abstract=291140)

[Mossin (1966)] Mossin, J., 1966, Equilibrium in capital asset market, Econometrica, 35, 768-783.

[Nelsen (1998)] Nelsen, R.B., 1998, An Introduction to Copulas. Lectures Notes in statistic 139 (Springer Verlag, New York).

[Pagan (1996)] Pagan, A., 1996, The Econometrics of Financial Markets, Journal of Empirical Finance, 3, 15 - 102.

[Patton (2001)] Patton, J.A., 2001, Estimation of copula models for time series of possibly different lengths, University of California, Econ. Disc. Paper No. 2001-17.

[Roll (1988)] Roll, R.,1988, The International Crash of October 1987, Financial Analysts Journal 19-35

[Ross (1976)] Ross, S., 1976, The arbitrage theory of capital asset pricing, Journal of Economic Theory, 17, 254-286.

[Sharpe (1964)] Sharpe, W., 1964, Capital assets prices: a theory of market equilibrium under conditions of risk, Journal of Finance, 19, 425-442.

[Sklar (1959)] Sklar, A., 1959, Fonction de répartition à n dimensions et leurs marges, Publ. Inst. Statist. Univ. Paris 8, 229-231.

[Vasicek (1977)] Vasicek, O., 1977, An equilibrium characterisation of the term structure of interest rates, Journal of Financial Economics, 5, 177-188.

Statistical description of the set of studied stocks

	July 1962 - December 1979				January 1980 - December 2000				July 1962 - December 2000			
	Mean	Std.	Skew.	Kurt.	Mean	Std.	Skew.	Kurt.	Mean	Std.	Skew.	Kurt.
Abbott Labs	0.6677	0.0154	0.2235	2.192	0.9217	0.0174	-0.0434	2.248	0.8066	0.0165	0.0570	2.300
American Home Products Corp.	0.4755	0.0136	0.2985	3.632	0.8486	0.0166	0.1007	8.519	0.6803	0.0154	0.1717	7.557
Boeing Co.	0.8460	0.0228	0.6753	4.629	0.7752	0.0193	0.1311	4.785	0.8068	0.0209	0.4495	4.901
Bristol-Myers Squibb Co.	0.5342	0.0152	-0.0811	2.808	0.9353	0.0175	-0.3437	16.733	0.7546	0.0165	-0.2485	12.573
Chevron Corp.	0.4916	0.0134	0.2144	2.442	0.6693	0.0169	0.0491	4.355	0.5885	0.0154	0.1033	4.209
Du Pont (E.I.) de Nemours & Co.	0.2193	0.0126	0.3493	2.754	0.6792	0.0172	-0.1021	4.731	0.4715	0.0153	0.0231	4.937
Disney (Walt) Co.	0.9272	0.0215	0.2420	2.762	0.8759	0.0195	-0.6661	17.655	0.8997	0.0204	-0.1881	9.568
General Motors Corp.	0.3547	0.0126	0.4138	4.302	0.5338	0.0183	-0.0128	5.373	0.4538	0.0160	0.0872	6.164
Hewlett-Packard Co.	0.7823	0.0199	0.0212	3.063	0.8913	0.0238	0.0254	4.921	0.8420	0.0221	0.0256	4.624
Coca-Cola Co.	0.4829	0.0138	0.0342	5.436	0.9674	0.0170	-0.1012	14.377	0.7483	0.0157	-0.0513	12.611
Minnesota Mining & MFG Co.	0.3459	0.0139	0.3016	2.997	0.6885	0.0150	-0.7861	20.609	0.5333	0.0145	-0.3550	14.066
Philip Morris Cos Inc.	0.7930	0.0153	0.2751	2.799	0.9664	0.0180	-0.2602	10.954	0.8863	0.0169	-0.0784	8.790
Pepsico Inc.	0.4982	0.0147	0.2380	2.867	0.9443	0.0180	0.1372	4.594	0.7431	0.0166	0.1786	4.413
Procter & Gamble Co.	0.3569	0.0115	0.3911	4.343	0.7916	0.0164	-1.6610	46.916	0.5947	0.0144	-1.2408	44.363
Pharmacia Corp.	0.3801	0.0145	0.2699	3.508	0.9027	0.0191	-0.6133	13.587	0.6666	0.0172	-0.3773	12.378
Schering-Plough Corp.	0.6328	0.0163	0.2619	3.112	1.0663	0.0192	0.1781	7.9979	0.8703	0.0179	0.2139	6.757
Texaco Inc.	0.3416	0.0134	0.2656	2.596	0.6644	0.0166	0.1192	6.477	0.5197	0.0152	0.1725	5.829
Texas Instruments Inc.	0.6839	0.0198	0.2076	3.174	1.0299	0.0268	0.1595	7.848	0.8726	0.0239	0.1831	7.737
United Technologies Corp	0.5801	0.0185	0.3397	2.826	0.7752	0.0170	0.0396	3.190	0.6876	0.0177	0.1933	3.034
Walgreen Co.	0.5851	0.0165	0.3530	3.030	1.1996	0.0185	0.1412	3.316	0.9217	0.0176	0.2260	3.295
Standart & Poor's 500	0.1783	0.0075	0.2554	3.131	0.5237	0.0101	-1.6974	36.657	0.3674	0.0090	-1.2236	32.406

Table 1: This table gives the main statistical features of the three samples we have considered. The columns *Mean*, *Std.*, *Skew.* and *Kurt.* respectively give the average return multiplied by one thousand, the standard deviation, the skewness and the excess kurtosis of each asset over the time intervals form July 1962 to December 1979, January 1980 to Decemeber 2000 and July 1962 to December 2000. The excess kurtosis is given as indicative of the relative weight of large return amplitudes, and can always be calculated over a finite time series even if it may not be asymptotically defined for power tails with exponents less than 4.

Estimation of the parameters of the factor model

	July 1962 - December 1979		January 1979 - December 2000		July 1962 - December 1979	
	β	ρ	β	ρ	β	ρ
Abbott Labs	0.9010	-0.0009	0.9145	-0.0016	0.9103	-0.0013
American Home Products Corp.	0.9865	-0.0006	0.8124	-0.0015	0.8668	-0.0011
Boeing Co.	1.4435	-0.0007	0.9052	-0.0009	1.0733	-0.0009
Bristol-Myers Squibb Co.	1.0842	-0.0006	1.0455	-0.0014	1.0576	-0.0011
Chevron Corp.	1.0072	-0.0007	0.8345	-0.0008	0.8885	-0.0008
Du Pont (E.I.) de Nemours & Co.	1.0819	-0.0001	0.9461	-0.0007	0.9885	-0.0004
Disney (Walt) Co.	1.5551	-0.0009	1.0034	-0.0011	1.1757	-0.0011
General Motors Corp.	1.0950	-0.0004	1.0112	0.0000	1.0374	-0.0002
Hewlett-Packard Co.	1.3926	-0.0008	1.3085	-0.0005	1.3348	-0.0008
Coca-Cola Co.	1.0357	-0.0006	0.9856	-0.0017	1.0012	-0.0012
Minnesota Mining & MFG Co.	1.1344	-0.0003	0.8768	-0.0010	0.9573	-0.0006
Philip Morris Cos Inc.	1.0913	-0.0011	0.8624	-0.0017	0.9339	-0.0015
Pepsico Inc.	0.9597	-0.0006	0.9028	-0.0016	0.9206	-0.0011
Procter & Gamble Co.	0.8299	-0.0005	0.8955	-0.0012	0.8750	-0.0009
Pharmacia Corp.	1.0756	-0.0004	0.8846	-0.0013	0.9443	-0.0009
Schering-Plough Corp.	1.1258	-0.0007	1.0506	-0.0017	1.0741	-0.0013
Texaco Inc.	1.4592	-0.0006	1.3826	-0.0007	1.4065	-0.0007
Texas Instruments Inc.	0.9419	-0.0004	0.6617	-0.0011	0.7492	-0.0007
United Technologies Corp	1.1348	-0.0005	0.9064	-0.0011	0.9777	-0.0009
Walgreen Co.	0.6369	-0.0007	0.8592	-0.0024	0.7898	-0.0016

Table 2: This table presents the estimated coefficient β for the factor model (6) and the correlation coefficient ρ between the factor and the estimated idiosyncratic noise, for the different time intervals we have considered. A Fisher's test shows that at the 95% confidence level none of the correlation coefficient is significantly different from zero.

Tail index for the time interval from July 1962 to December 1979

	Negative Tail						Positive Tail					
	q = 1%		q = 2.5%		q = 5%		q = 1%		q = 2.5%		q = 5%	
	Asset	ε	Asset	ε	Asset	ε	Asset	ε	Asset	ε	Asset	ε
Abbott Labs	5.54	5.31	3.94	4.02	3.27	3.31	5.10	4.50	4.09	3.71	3.53*	3.14
American Home Products Corp.	4.58	5.11	3.89	3.81	3.02*	3.21*	3.64	4.66	3.60	3.81	3.11	3.15
Boeing Co.	6.07	4.90	4.57	3.74	3.32	3.49	4.04	4.27	3.95	4.19	3.35*	2.93
Bristol-Myers Squibb Co.	4.32	4.27	3.31	3.95	2.99*	3.16*	5.96*	5.19	3.94	4.82*	3.62*	4.03*
Chevron Corp.	5.24	4.78	3.75	3.29	2.91	3.12*	5.21	5.15	3.90	4.26	3.25*	3.07
Du Pont (E.I.) de Nemours & Co.	5.26	4.36	3.69	3.76	3.17*	3.23*	5.35	5.15	4.00	3.37	3.13	3.04
Disney (Walt) Co.	3.59	4.23	3.59	3.84	3.08*	3.22*	4.90	4.34	4.26	3.73	3.33*	3.29*
General Motors Corp.	4.82	3.50	3.72	3.66	2.94*	3.36	3.91	4.78	3.64	3.86	2.94	3.07
Hewlett-Packard Co.	3.76	3.89	3.12*	3.05*	2.81*	3.00*	4.64	5.08	4.08	4.20	3.41*	3.42*
Coca-Cola Co.	3.45	3.45	3.05*	3.71	2.75*	3.17*	3.91	4.26	3.16	3.61	2.81	3.16
Minnesota Mining & MFG Co.	5.16	4.86	4.06	4.35	3.43	3.71	4.35	4.47	3.96	3.31	3.14	3.06
Philip Morris Cos Inc.	4.63	3.79	3.82	3.90	3.38	3.48	4.10	4.64	3.59	3.85	3.03	3.06
Pepsico Inc.	4.89	5.35	3.93	4.49	3.02*	3.27	4.07	4.67	3.49	3.86	3.15	3.21*
Procter & Gamble Co.	4.42	3.77	3.77	3.74	3.13*	3.42	4.14	5.39	3.59	3.73	2.97	3.40*
Pharmacia Corp.	4.73	4.24	4.05	3.45	2.88*	3.34	4.46	3.72	3.95	3.90	3.14	2.99
Schering-Plough Corp.	4.59	4.70	4.20	3.87	3.37	3.33	4.60	5.88*	3.50	3.91	3.07	3.22*
Texaco Inc.	5.34	4.59	3.99	3.84	3.07*	3.19*	3.83	4.10	3.94	3.67	3.14	2.98
Texas Instruments Inc.	4.08	4.54	3.36	3.13*	3.22*	2.87*	4.52	4.20	3.67	3.79	3.16	3.07
United Technologies Corp	4.00	4.49	3.52	3.92	3.27	3.46	4.78	4.97	3.73	3.98	3.26*	3.49*
Walgreen Co.	4.63	6.50	3.85	4.26	2.94*	3.18*	5.16	4.56	3.47	3.30	3.15	2.82
Standart & Poor's 500	5.17	-	4.16	-	3.91	-	3.74	-	3.34	-	2.64	-

Table 3: This table gives the estimated value of the tail index for the twenty considered assets, the Standard & Poor's 500 index and the residues obtained by regressing each asset on the Standard & Poor's 500 index, for both the negative and the positive tails, during the time interval from July 1962 to December 1979. The tail indexes are estimed by the Hill's estimator at the quantile 1%, 2.5% and 5% which are the optimal quantiles given by the [Hall (1990)] and [Danielsson and de Vries (1997)]'s algorithms. The values decorrated with stars represent the tail indexes which cannot be considered equal to the Standard & Poor's 500 index's tail index at the 95% confidence level.

Tail index for the time interval from January 1980 to December 2000

	Negative Tail						Positive Tail					
	q = 1%		q = 2.5%		q = 5%		q = 1%		q = 2.5%		q = 5%	
	Asset	ε	Asset	ε	Asset	ε	Asset	ε	Asset	ε	Asset	ε
Abbott Labs	3.59	3.60	3.35	3.62	3.22	3.39	5.14	4.60	4.16	3.76	3.77	3.07
American Home Products Corp.	3.03	3.07	3.11	2.78	2.73	2.49*	4.01	3.47	3.28	3.02	2.87	2.79
Boeing Co.	3.39	3.97	3.23	3.53	3.02	3.21	4.86	3.65	3.45	3.16	3.13	3.23
Bristol-Myers Squibb Co.	3.21	3.15	2.90	3.41	2.80	3.16	2.98	3.74	3.35	3.12	3.20	2.75
Chevron Corp.	4.13	4.48	3.99	3.91	3.30	3.45	5.16	4.53	3.88	3.81	3.01	3.06
Du Pont (E.I.) de Nemours & Co.	3.99	3.49	3.76	3.23	3.02	3.04	5.36	4.33	4.31	3.35	3.44	2.76
Disney (Walt) Co.	2.83	3.24	2.76	2.97	2.85	2.83	3.97	3.70	3.68	3.33	3.15	2.87
General Motors Corp.	4.44	4.79	3.88	4.27*	3.44	3.56	5.76	5.32	4.45	3.86	3.43	3.22
Hewlett-Packard Co.	3.73	3.45	3.52	3.12	3.00	2.73	4.31	3.40	3.47	3.29	3.24	2.99
Coca-Cola Co.	3.01	3.76	3.14	3.48	2.99	2.86	4.06	3.47	3.45	3.16	3.37	2.87
Minnesota Mining & MFG Co.	3.52	3.38	3.21	3.39	2.88	3.04	3.76	3.46	3.95	3.22	3.10	2.76
Philip Morris Cos Inc.	3.58	3.34	3.33	3.12	2.68	2.53*	3.42	3.16	3.70	3.07	2.85	2.81
Pepsico Inc.	4.14	4.46	3.39	3.60	2.99	3.27	4.00	3.87	3.61	3.34	3.44	3.31
Procter & Gamble Co.	2.65	2.46	3.29	3.19	3.19	2.87	4.35	3.90	3.48	3.20	3.14	2.91
Pharmacia Corp.	2.96	3.20	3.09	2.79	2.80	2.70	4.12	4.70	3.44	3.50	3.31	2.89
Schering-Plough Corp.	4.22	5.20*	3.29	3.68	3.11	3.05	3.23	3.51	3.45	3.08	3.06	2.87
Texaco Inc.	3.09	3.20	3.10	3.15	2.88	2.84	3.65	3.36	3.20	3.04	2.86	2.70
Texas Instruments Inc.	3.49	3.53	3.35	3.31	2.89	2.99	4.00	3.42	3.36	3.30	2.97	3.06
United Technologies Corp	4.21	3.98	3.82	3.46	3.34	3.18	5.39	4.50	4.00	3.80	3.51	3.26
Walgreen Co.	4.06	4.35	3.81	4.04	3.20	3.40	4.60	5.12	3.79	3.54	3.20	3.07
Standart & Poor's 500	3.16	-	3.17	-	3.16	-	4.00	-	3.65	-	3.19	-

Table 4: This table gives the estimated value of the tail index for the twenty considered assets, the Standard & Poor's 500 index and the residues obtained by regressing each asset on the Standard & Poor's 500 index, for both the negative and the positive tails, during the time interval from January 1980 to December 2000. The tail indexes are estimed by the Hill's estimator at the quantile 1%, 2.5% and 5% which are the optimal quantiles given by the [Hall (1990)] and [Danielsson and de Vries (1997)]'s algorithms. The values decorrated with stars represent the tail indexes whose value cannot be considered equal to the Standard & Poor's 500 index's tail index at the 95% confidence level.

Coefficient of lower tail dependence during the time interval from July 1962 to December 2000 for a tail index equal to three

	First Centile				First Quintile				First Decile			
	mean	std.	min.	max.	mean	std.	min.	max.	mean	std.	min.	max.
Abbott Labs	0.1670	0.0127	0.1442	0.2137	0.1633	0.0071	0.1442	0.2137	0.1540	0.0120	0.1331	0.2137
American Home Products Corp.	0.1423	0.0207	0.0910	0.1720	0.1728	0.0205	0.091	0.1963	0.1823	0.0175	0.0910	0.2020
Boeing Co.	0.1372	0.0127	0.1101	0.1804	0.1349	0.0064	0.1101	0.1804	0.1289	0.0078	0.1101	0.1804
Bristol-Myers Squibb Co.	0.2720	0.0231	0.1878	0.3052	0.2751	0.0115	0.1878	0.3052	0.2696	0.0110	0.1878	0.3052
Chevron Corp.	0.1853	0.0188	0.1656	0.2564	0.1790	0.0105	0.1634	0.2564	0.1748	0.0096	0.1606	0.2564
Du Pont (E.I.) de Nemours & Co.	0.2547	0.0148	0.2127	0.2871	0.2695	0.0117	0.2127	0.2876	0.2685	0.0103	0.2127	0.2876
Disney (Walt) Co.	0.1772	0.0149	0.1368	0.1957	0.1938	0.0123	0.1368	0.2094	0.1900	0.0109	0.1368	0.2094
General Motors Corp.	0.2641	0.0259	0.2393	0.3652	0.2565	0.0138	0.2349	0.3652	0.2545	0.0108	0.2349	0.3652
Hewlett-Packard Co.	0.1701	0.0096	0.1389	0.1914	0.2018	0.0230	0.1389	0.2303	0.2039	0.0176	0.1389	0.2303
Coca-Cola Co.	0.2343	0.0223	0.1686	0.2719	0.2576	0.0163	0.1686	0.2731	0.2579	0.0123	0.1686	0.2731
Minnesota Mining & MFG Co.	0.2844	0.0196	0.2399	0.3407	0.2873	0.0099	0.2399	0.3407	0.2802	0.0117	0.2399	0.3407
Philip Morris Cos Inc.	0.1369	0.0168	0.0983	0.1673	0.1700	0.0206	0.0983	0.1919	0.1729	0.0155	0.0983	0.1919
Pepsico Inc.	0.1634	0.0132	0.1483	0.2106	0.1535	0.0083	0.1448	0.2106	0.1512	0.0067	0.1434	0.2106
Procter & Gamble Co.	0.2284	0.0292	0.1434	0.2673	0.2461	0.0169	0.1434	0.2673	0.2413	0.0141	0.1434	0.2673
Pharmacia Corp.	0.1279	0.0104	0.0863	0.1432	0.1588	0.0192	0.0863	0.1822	0.1643	0.0149	0.0863	0.1822
Schering-Plough Corp.	0.2195	0.0190	0.1920	0.2863	0.2179	0.0103	0.1920	0.2863	0.2107	0.0123	0.1877	0.2863
Texaco Inc.	0.4355	0.0195	0.3389	0.4906	0.4500	0.0142	0.3389	0.4906	0.4515	0.011	0.3389	0.4906
Texas Instruments Inc.	0.0327	0.0027	0.0243	0.0369	0.0369	0.0033	0.0243	0.0414	0.0371	0.0027	0.0243	0.0414
United Technologies Corp	0.1570	0.0153	0.1298	0.2182	0.1562	0.0075	0.1298	0.2182	0.1511	0.0084	0.1298	0.2182
Walgreen Co.	0.0937	0.0112	0.0808	0.1384	0.0837	0.0071	0.0776	0.1384	0.0786	0.0078	0.0669	0.1384

Table 5: This table gives the average (*mean*), the standard deviation (*std.*), the minimum (*min.*) and the maximum (*max.*) values of the coefficient of lower tail dependence estimated over the first centile, quintile and decile during the entire time interval from July 1962 to December 2000, under the assumption that the tail index equals three.

Coefficients of tail dependence during the time interval from July 1962 to December 1979

	Negative Tail			Positive Tail		
	$\alpha = 3$	$\alpha = 3.5$	$\alpha = 4$	$\alpha = 3$	$\alpha = 3.5$	$\alpha = 4$
Abbott Labs	0.12	0.09	0.06	0.11	0.08	0.06
American Home Products Corp.	0.22	0.18	0.15	0.25	0.22	0.19
Boeing Co.	0.16	0.13	0.10	0.13	0.10	0.07
Bristol-Myers Squibb Co.	0.22	0.19	0.16	0.28	0.25	0.23
Chevron Corp.	0.21	0.17	0.14	0.26	0.23	0.20
Du Pont (E.I.) de Nemours & Co.	0.38	0.37	0.35	0.37	0.35	0.33
Disney (Walt) Co.	0.24	0.20	0.17	0.23	0.19	0.16
General Motors Corp.	0.39	0.37	0.35	0.48	0.47	0.47
Hewlett-Packard Co.	0.15	0.12	0.09	0.23	0.20	0.17
Coca-Cola Co.	0.26	0.22	0.19	0.26	0.23	0.20
Minnesota Mining & MFG Co.	0.35	0.32	0.30	0.35	0.33	0.31
Philip Morris Cos Inc.	0.25	0.22	0.19	0.20	0.17	0.14
Pepsico Inc.	0.15	0.12	0.09	0.17	0.14	0.11
Procter & Gamble Co.	0.23	0.19	0.16	0.24	0.21	0.18
Pharmacia Corp.	0.23	0.19	0.16	0.26	0.23	0.20
Schering-Plough Corp.	0.21	0.18	0.15	0.20	0.17	0.14
Texaco Inc.	0.47	0.46	0.46	0.49	0.49	0.49
Texas Instruments Inc.	0.06	0.04	0.03	0.07	0.05	0.03
United Technologies Corp	0.13	0.10	0.07	0.13	0.10	0.07
Walgreen Co.	0.03	0.02	0.01	0.02	0.01	0.01

Table 6: This table summarizes the mean values over the first centile of the distribution of the coefficients of (upper or lower) tail dependence for the positive and negative tails during the time interval from July 1962 to December 1979, for three values of the tail index $\alpha = 3$, 3.5, 4.

Coefficients of tail dependence during the time interval from January 1980 to December 2000

	Negative Tail			Positive Tail		
	$\alpha = 3$	$\alpha = 3.5$	$\alpha = 4$	$\alpha = 3$	$\alpha = 3.5$	$\alpha = 4$
Abbott Labs	0.20	0.17	0.14	0.16	0.13	0.10
American Home Products Corp.	0.12	0.09	0.06	0.10	0.08	0.05
Boeing Co.	0.14	0.11	0.08	0.10	0.07	0.05
Bristol-Myers Squibb Co.	0.32	0.29	0.26	0.25	0.21	0.19
Chevron Corp.	0.18	0.14	0.11	0.13	0.09	0.07
Du Pont (E.I.) de Nemours & Co.	0.23	0.20	0.17	0.16	0.13	0.10
Disney (Walt) Co.	0.16	0.13	0.10	0.15	0.12	0.09
General Motors Corp.	0.26	0.22	0.19	0.20	0.16	0.13
Hewlett-Packard Co.	0.19	0.15	0.13	0.21	0.18	0.15
Coca-Cola Co.	0.24	0.20	0.18	0.20	0.17	0.14
Minnesota Mining & MFG Co.	0.26	0.23	0.20	0.20	0.17	0.14
Philip Morris Cos Inc.	0.11	0.08	0.06	0.11	0.08	0.06
Pepsico Inc.	0.17	0.14	0.11	0.14	0.11	0.09
Procter & Gamble Co.	0.24	0.21	0.18	0.20	0.16	0.13
Pharmacia Corp.	0.10	0.08	0.05	0.10	0.07	0.05
Schering-Plough Corp.	0.23	0.20	0.17	0.16	0.13	0.10
Texaco Inc.	0.43	0.42	0.41	0.31	0.28	0.26
Texas Instruments Inc.	0.02	0.01	0.01	0.02	0.01	0.01
United Technologies Corp	0.20	0.16	0.14	0.18	0.14	0.11
Walgreen Co.	0.15	0.12	0.09	0.09	0.07	0.05

Table 7: This table summarizes the mean values over the first centile of the distribution of the coefficients of (upper or lower) tail dependence for the positive and negative tails during the time interval from January 1980 to December 2000, for three values of the tail index $\alpha =$ 3, 3.5, 4.

Coefficients of tail dependence during the time interval from July 1962 to December 2000

	Negative Tail			Positive Tail		
	$\alpha = 3$	$\alpha = 3.5$	$\alpha = 4$	$\alpha = 3$	$\alpha = 3.5$	$\alpha = 4$
Abbott Labs	0.17	0.13	0.11	0.15	0.12	0.09
American Home Products Corp.	0.14	0.11	0.08	0.15	0.11	0.09
Boeing Co.	0.14	0.10	0.08	0.10	0.07	0.05
Bristol-Myers Squibb Co.	0.27	0.24	0.21	0.27	0.24	0.21
Chevron Corp.	0.19	0.15	0.12	0.17	0.13	0.10
Du Pont (E.I.) de Nemours & Co.	0.25	0.22	0.19	0.23	0.19	0.16
Disney (Walt) Co.	0.18	0.14	0.11	0.17	0.13	0.11
General Motors Corp.	0.26	0.23	0.20	0.24	0.21	0.18
Hewlett-Packard Co.	0.17	0.14	0.11	0.23	0.19	0.16
Coca-Cola Co.	0.23	0.20	0.17	0.23	0.20	0.17
Minnesota Mining & MFG Co.	0.28	0.25	0.23	0.25	0.22	0.19
Philip Morris Cos Inc.	0.14	0.10	0.08	0.14	0.11	0.08
Pepsico Inc.	0.16	0.13	0.10	0.16	0.12	0.10
Procter & Gamble Co.	0.23	0.20	0.17	0.22	0.18	0.15
Pharmacia Corp.	0.13	0.10	0.07	0.14	0.10	0.08
Schering-Plough Corp.	0.22	0.19	0.16	0.19	0.15	0.12
Texaco Inc.	0.44	0.42	0.41	0.37	0.35	0.33
Texas Instruments Inc.	0.03	0.02	0.01	0.03	0.02	0.01
United Technologies Corp	0.16	0.12	0.10	0.15	0.12	0.09
Walgreen Co.	0.09	0.07	0.05	0.06	0.04	0.03

Table 8: This table summarizes the mean values over the first centile of the distribution of the coefficients of (upper or lower) tail dependence for the positive and negative tails during the time interval from July 1962 to December 2000, for three values of the tail index $\alpha = 3$, 3.5, 4.

Comparison of the estimated coefficient of lower tail dependence with the realized extreme losses

	July 1962 - Dec. 1979			Jan.1980 - Dec. 2000		
	Extremes	λ_-	p-value	Extremes	λ_-	p-value
Abbott Labs	0	0.12	0.2937	4	0.20	0.0904
American Home Products Corp.	1	0.22	0.2432	2	0.12	0.2247
Boeing Co.	0	0.16	0.1667	3	0.14	0.1176
Bristol-Myers Squibb Co.	2	0.22	0.2987	4	0.32	0.2144
Chevron Corp.	3	0.21	0.2112	4	0.18	0.0644
Du Pont (E.I.) de Nemours & Co.	0	0.38	0.0078	4	0.23	0.1224
Disney (Walt) Co.	2	0.24	0.2901	2	0.16	0.2873
General Motors Corp.	2	0.39	0.1345	4	0.26	0.1522
Hewlett-Packard Co.	0	0.15	0.1909	2	0.19	0.3007
Coca-Cola Co.	2	0.26	0.2765	5	0.24	0.0494
Minnesota Mining & MFG Co.	2	0.35	0.1784	4	0.26	0.1571
Philip Morris Cos Inc.	1	0.25	0.1841	2	0.11	0.2142
Pepsico Inc.	2	0.15	0.2795	5	0.17	0.0141
Procter & Famble Co.	1	0.23	0.2245	3	0.24	0.2447
Pharmacia Corp.	2	0.23	0.2956	4	0.10	0.0128
Schering-Plough Corp.	0	0.21	0.0946	4	0.23	0.1224
Texaco Inc.	1	0.47	0.0161	3	0.43	0.1862
Texas Instruments Inc.	0	0.06	0.5222	2	0.02	0.0212
United Technologies Corp	1	0.13	0.3728	4	0.20	0.0870
Walgreen Co.	1	0.03	0.2303	3	0.15	0.1373

Table 9: This table gives, for the time intervals from July 1962 to December 1979 and from January 1980 to December 2000, the number of losses within the ten largest losses incured by an asset which have occured together with one of the ten largest losses of the Standard & Poor's 500 index during the same time interval. The probabilty of occurence of such a realisation is given by the p-value derived from the binomial law (15) with parameter λ_-.

Comparison of the estimated coefficient of lower tail dependence with the realized non-comonotonic extreme losses

	July 1962 - Dec. 1979			Jan.1980 - Dec. 2000		
	Extremes	λ_-	p-value	Extremes	λ_-	p-value
Abbott Labs	0	0.12	0.2937	4	0.20	0.0904
American Home Products Corp.	1	0.22	0.2432	1	0.12	0.3828
Boeing Co.	0	0.16	0.1667	3	0.14	0.1176
Bristol-Myers Squibb Co.	2	0.22	0.2987	3	0.32	0.2653
Chevron Corp.	3	0.21	0.2112	3	0.18	0.1708
Du Pont (E.I.) de Nemours & Co.	0	0.38	0.0078	3	0.23	0.2342
Disney (Walt) Co.	2	0.24	0.2901	1	0.16	0.3300
General Motors Corp.	2	0.39	0.1345	3	0.26	0.2536
Hewlett-Packard Co.	0	0.15	0.1909	1	0.19	0.2880
Coca-Cola Co.	1	0.26	0.1782	4	0.24	0.1318
Minnesota Mining & MFG Co.	2	0.35	0.1784	3	0.26	0.2561
Philip Morris Cos Inc.	1	0.25	0.1841	2	0.11	0.2142
Pepsico Inc.	2	0.15	0.2795	5	0.17	0.0141
Procter & Famble Co.	1	0.23	0.2245	3	0.24	0.2447
Pharmacia Corp.	2	0.23	0.2956	4	0.10	0.0128
Schering-Plough Corp.	0	0.21	0.0946	3	0.23	0.2342
Texaco Inc.	1	0.47	0.0161	3	0.43	0.1862
Texas Instruments Inc.	0	0.06	0.5222	1	0.02	0.1922
United Technologies Corp	1	0.13	0.3728	3	0.20	0.2001
Walgreen Co.	1	0.03	0.2303	3	0.15	0.1373

Table 10: This table gives, for the time intervals from July 1962 to December 1979 and from January 1980 to December 2000, the number of losses within the ten largest losses incured by an asset which have occured together with one of the ten largest losses of the Standard & Poor's 500 index during the same time interval, provided that the losses are not both the largest of each series. The probabilty of occurence of such a realisation is given by the p-value derived from the binomial law (15) with parameter λ_-.